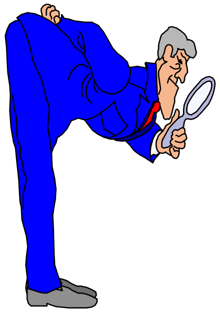
ADDITIONAL INFORMATION

TESTING:



- Weibull Analysis
- Data Analysis with Suspended Testing
- Confidence Levels

(12 addl)

WEIBULL DATA ANALYSIS

OBJECTIVES:

- **Understand the parameters associated with WD.**
- **When should the WD be used?**
- **Be able to analyze data with the WD.**
- **Understand confidence limits; when do they apply?**
- **Why are confidence limits important?**
- **How is data from suspended tests analyzed?**

BACKGROUND

- Developed by Waloddi Weibull (1887-1979)
- Interested in statistical distributions of material strength.
- His proposed distributions for material strength became known as WD.

DEFINITIONS

- **Weibull Distribution:**
- **Shape parameter (beta), β: the slope of the weibull cumulative distribution function.**
- **Scale parameter (eta), η: the “compression” of the weibull probability density function.**
- **Location parameter (gamma), γ: the “x” intercept of the weibull probability density function.**

WHY IMPORTANT

- One of the most widely used distributions.
- Highly flexible.
- Best fits many real world applications:
  - The WD represents the life of components and parts whereas the ED represents the life of assemblies and systems.
  - Mechanical components: ball bearings, motors, fatigue failure of some simple structures.
  - Failures where chemical actions are a predominant mechanism.

WHEN SHOULD THE WEIBULL DISTRIBUTION BE USED?

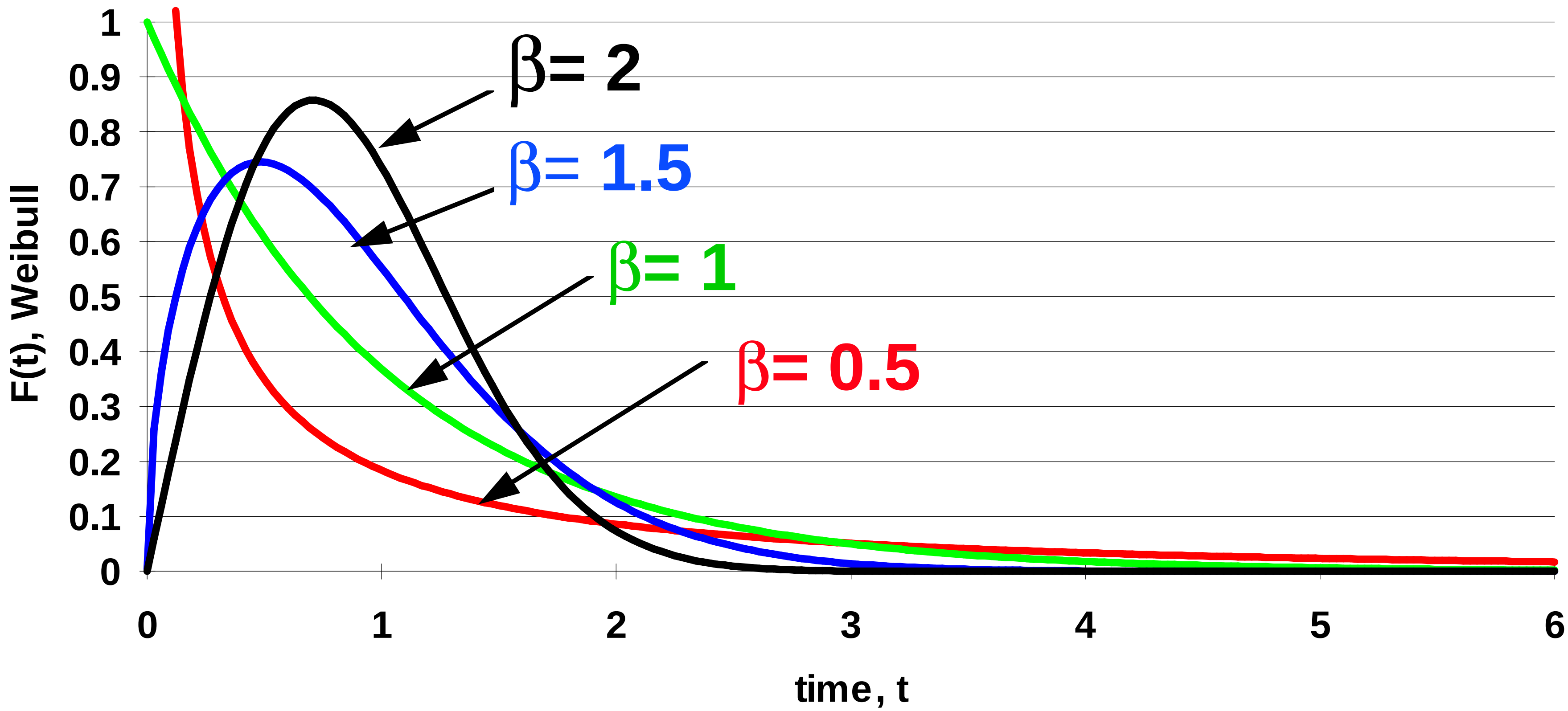
- **The WD should be used at or below the component level**
- **The WD should be used only when a single failure mode is expected.**
- **The WD is typically applied in analyzing mechanical failures.**
- **Analysis of components from more than one lot (unless the manufacturing process is carefully controlled) should be avoided.**

WEIBULL pdf

$$f(t)=(\beta/\eta)\,[(t-\gamma)/\eta]^{(\beta-1)}\,exp\,\{-[(t-\gamma)/\eta]^{\beta}\}$$

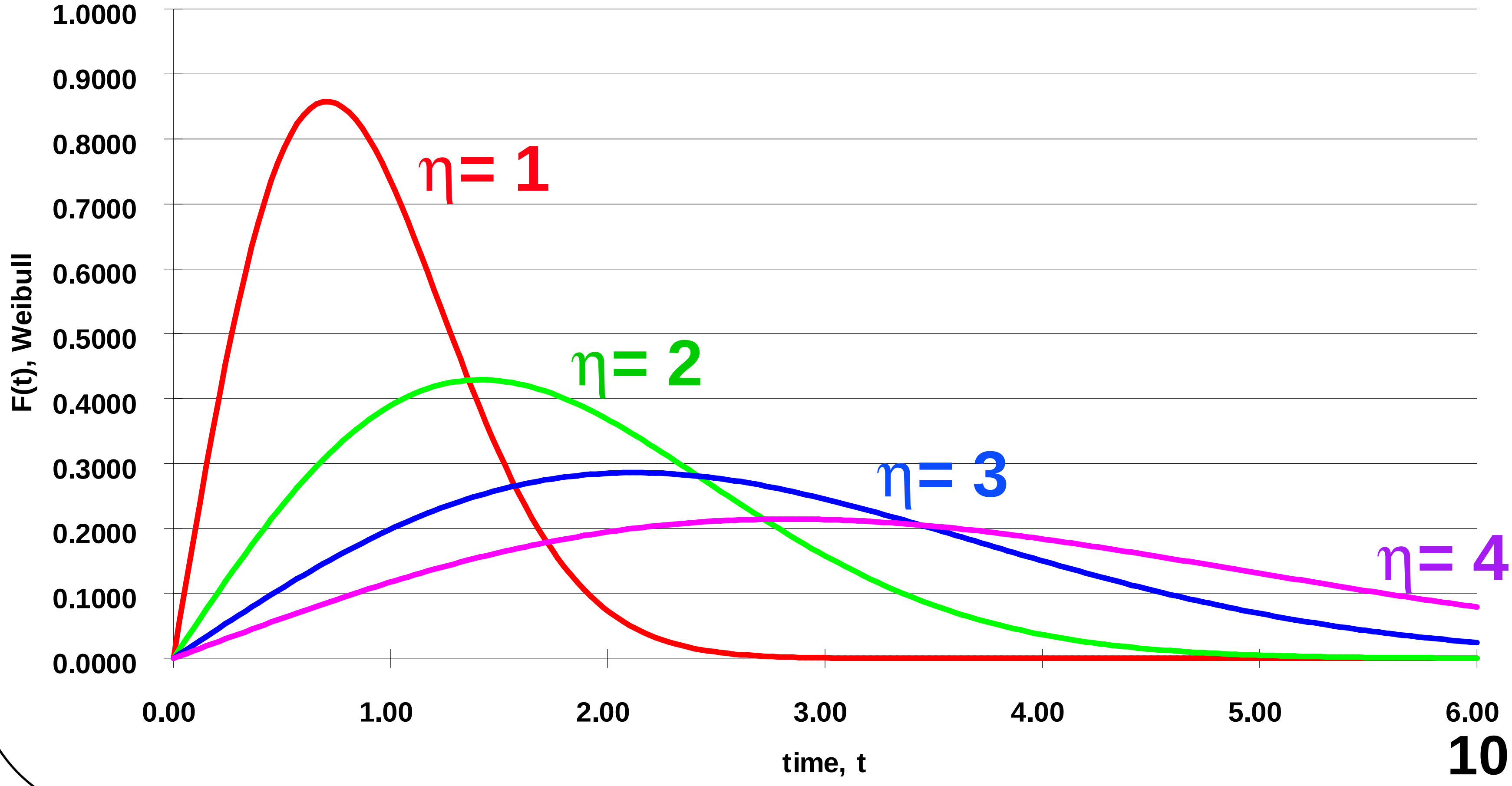
Weibull pdf

β = 0.5, 1, 1.5, 2, η =1,γ =0



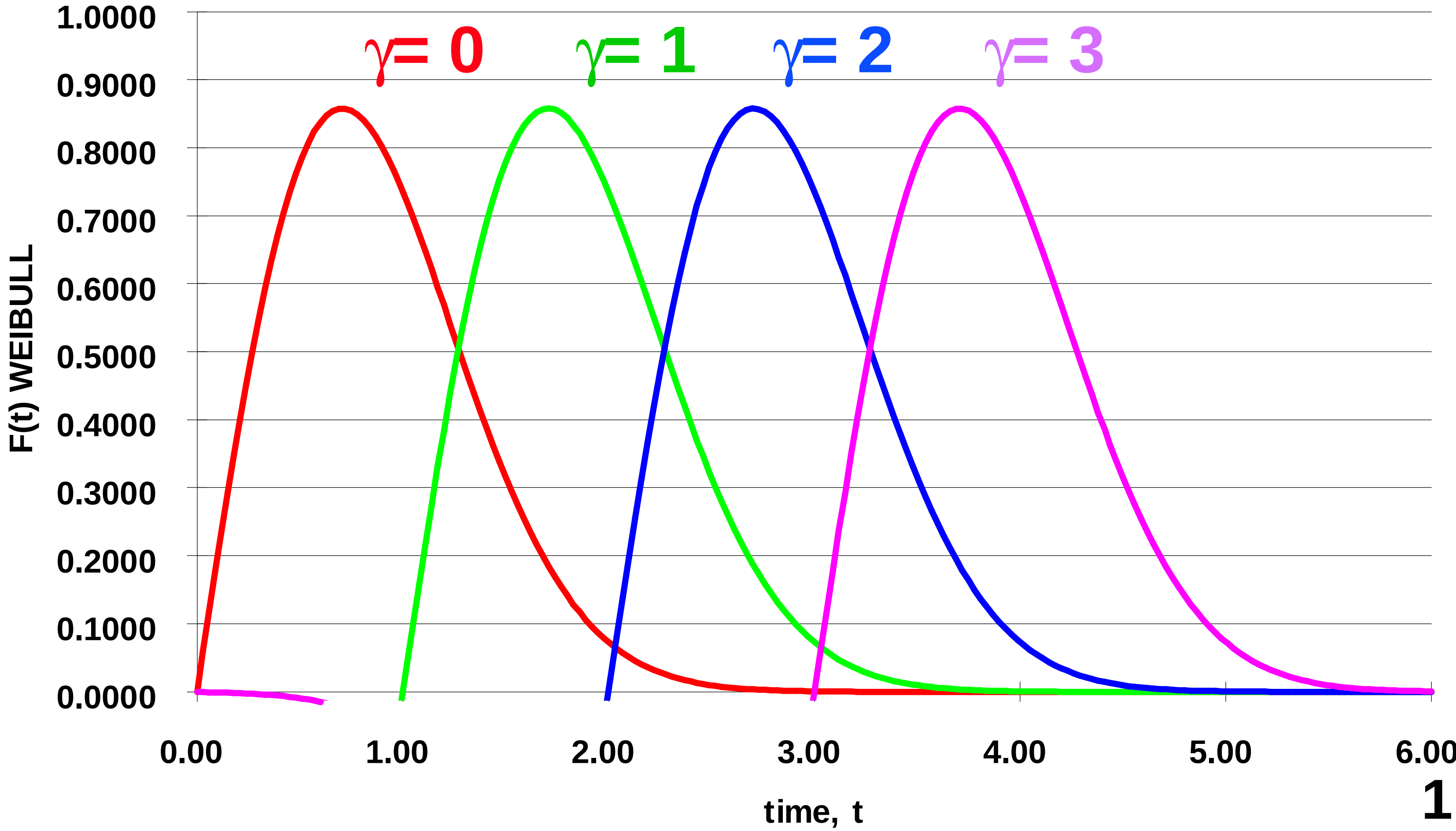
Weibull pdf

β = 2, η =1,2,3,4, γ =0



Weibull pdf

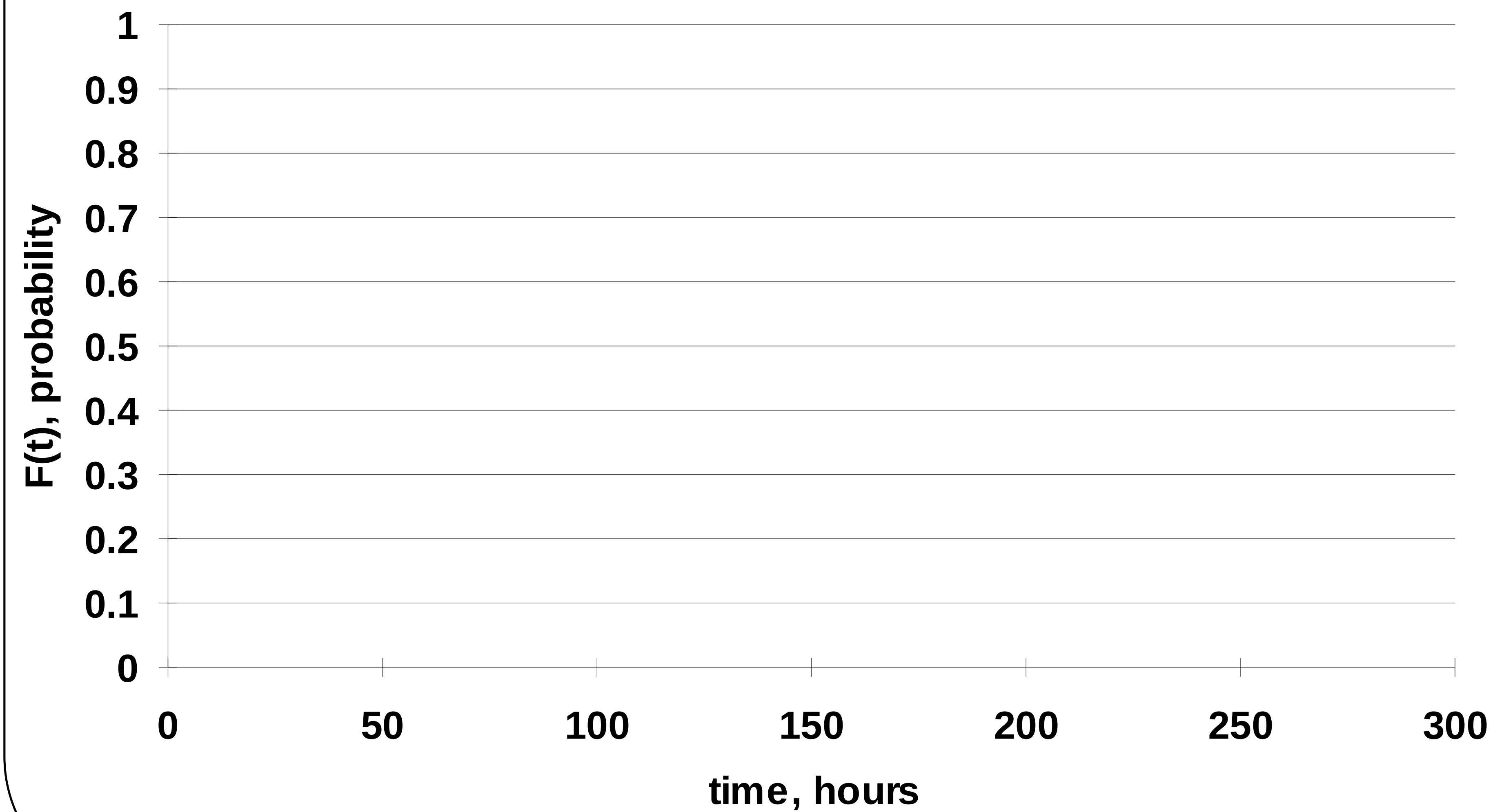
β = 2, η = 1, γ = 0,1,2,3



Example: **The failure time of 10 CONTROL SHUTTLES (CS-113) are recorded to be 85, 120, 145, 165, 185, 200, 220, 240, 260, 295 hrs. Use $F(t_i) = (i-0.3)/(n+0.4)$.**

i	*t*	$F(t_i)$	$R(t_i)$	$h(t)$	
1	85	____________	____________	____________	____________
2	120	____________	____________	____________	____________
3	145	____________	____________	____________	____________
4	____________	____________	____________	____________	____________
5	____________	____________	____________	____________	____________
6	____________	____________	____________	____________	____________
7	____________	____________	____________	____________	____________
8	____________	____________	____________	____________	____________
9	____________	____________	____________	____________	____________
10	____________	____________	____________	____________	____________

Example (con't) -- PLOT CDF & RF



F; P12-3 p. 2

PLOT ON WEIBULL PROBABILITY PAPER

F; P12-3 p.3

Attribute Testing & Confidence Levels

- **A reliability has associated with it a confidence level that the reliability number falls within a certain range. Thus we can say we are 95% confident that the reliability is 0.90. This is based on the sample size or number of tests.**
- **A confidence level shows the likelihood that a statistical estimate will coincide with the actual population value.**
- **As the sample size increases the statistical estimate becomes increasingly more accurate.**
- **Selection of the confidence level is a customer's or engineers choice and depends on the amount of risk they are willing to take on being wrong about the reliability of the device.**

Confidence Levels - EXAMPLE

- For a system with two components a fifty percent confidence level that both items will succeed is determined by:
- $R^2 + 2RQ + Q^2 = 1$ where
- R^2 = probability that both devices will pass
- $2RQ$ = probability that one device will pass
- Q^2 = probability that both devices will fail
- Assume a 50% probability that both units will pass.
- $R^2 = 0.50$ and $R = (0.50)^{1/2} = 0.71$
- There is a 50% confidence that the reliability of the device is 0.71

Confidence Levels - EXAMPLE II

- **Ten samples were tested and there was one observed failure. What is the predicted or demonstrated reliability at 90-percent confidence? Since for 1 failure we have:**

$$R^n + nR^{n-1}Q = (1 - \text{Confidence level})$$

- **For 10 samples we have:**

$$R^{10} + 10R^9Q = (1 - .90)$$

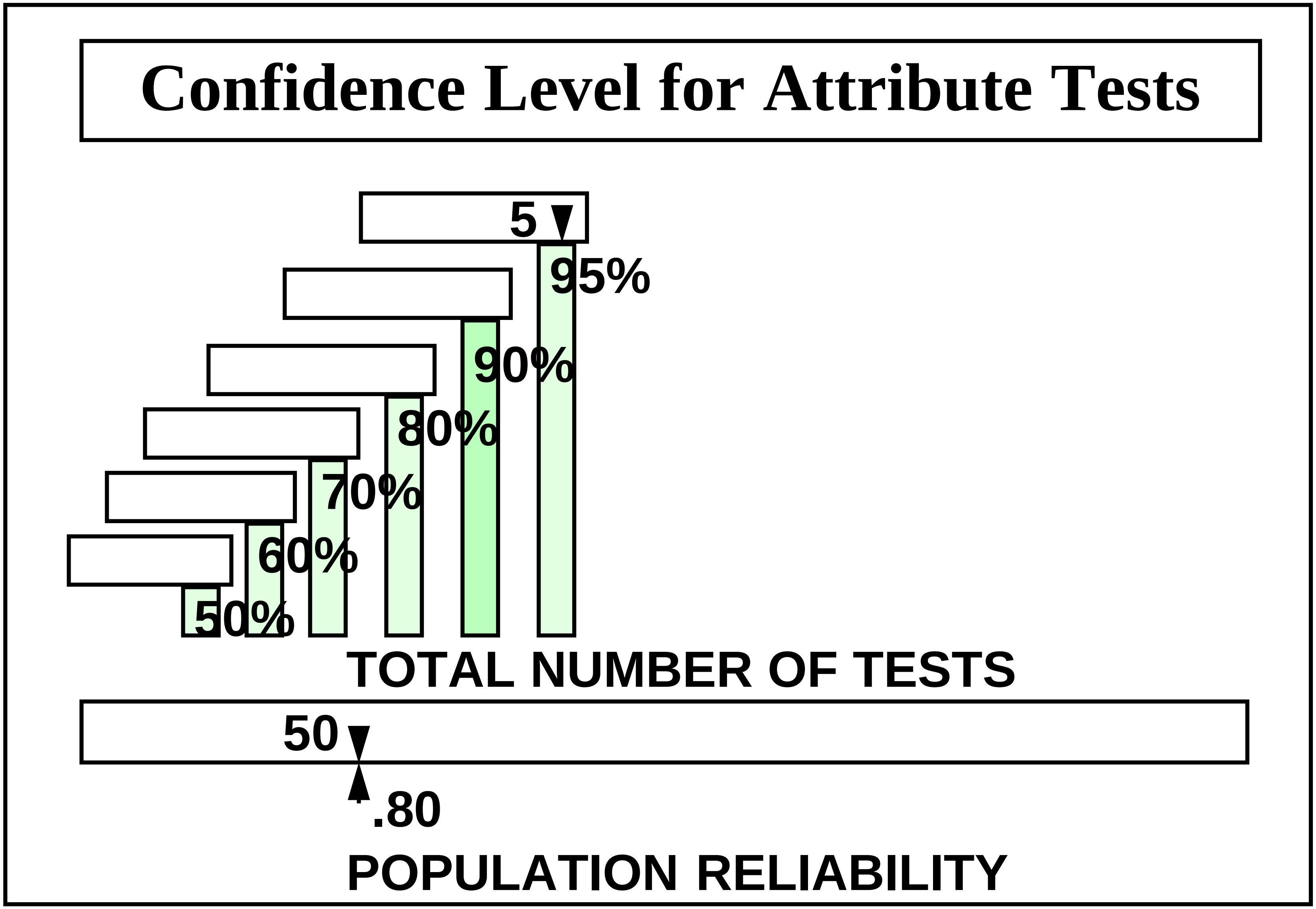
- $R = 0.663$

Application

- During flight testing of 50 missiles, five failures occur. What confidence do we have that the missile is 80% reliable?
- From the FIGURE A-4(f) p. 141 at 80% reliability and 50 events the confidence is 95%.
- Slide Rule: 5 FAILURES, 80% RELIABLE GIVES 95% confidence:

Confidence Level for Attribute Tests

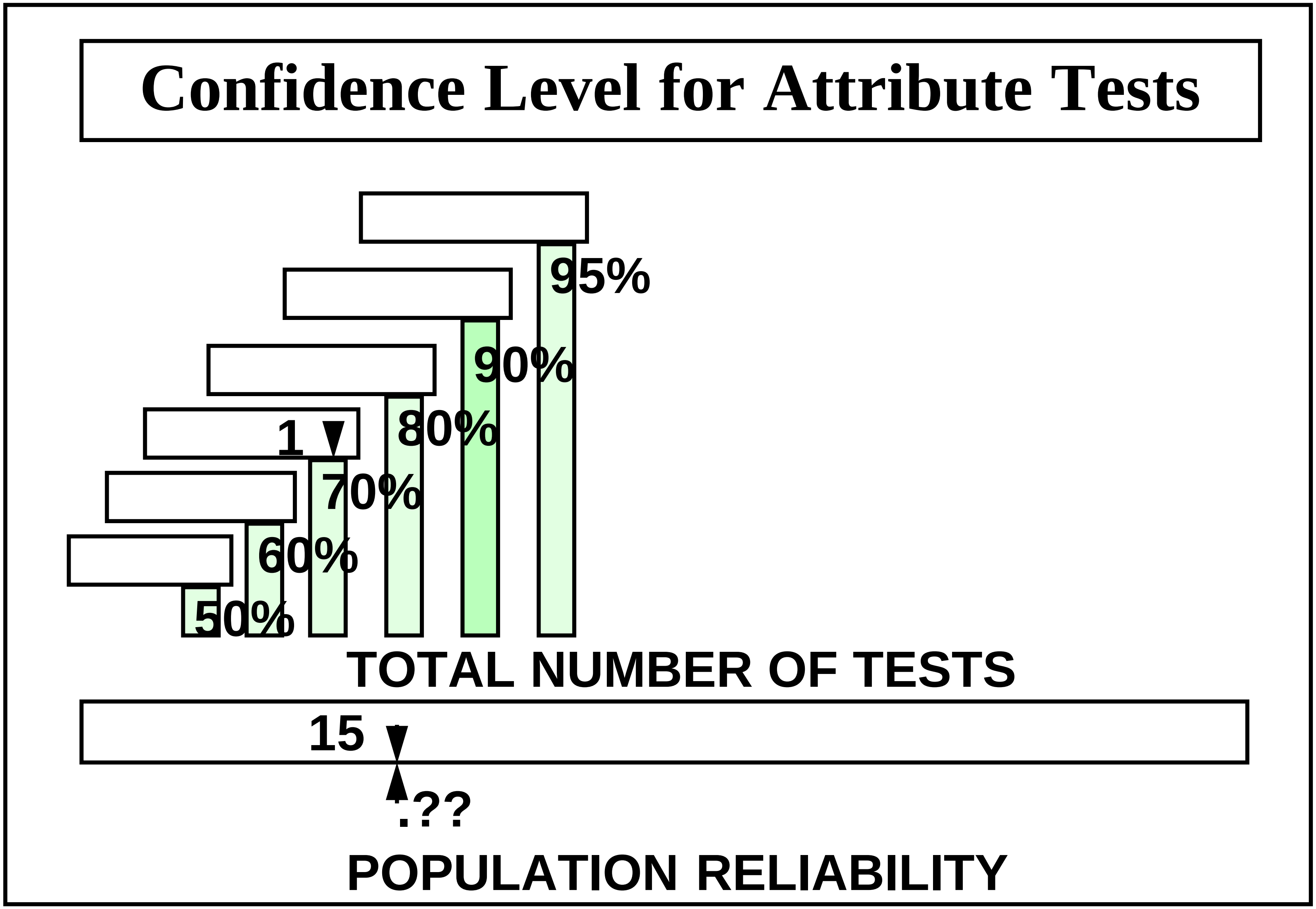
Slide Rule


Confidence Level for Attribute Tests
5
95%
90%
80%
70%
60%
50%
TOTAL NUMBER OF TESTS
50
.80
POPULATION RELIABILITY

Confidence Level for Attribute Test Slide Rule Example

- The statistical basis for this rule is the χ^2 approximation of the binomial distribution.
- 15 items are tested and one fails. What is the population reliability at 70% confidence?
- Use the Confidence Level for Attribute Test side and set the rule to 1 above the 70% confidence.
- Read from the TOTAL NUMBER OF TESTS = 15 on the scale for POPULATION RELIABILITY: This gives _____ population reliability.

Slide Rule


Confidence Level for Attribute Tests
95%
90%
80%
1
70%
60%
50%
TOTAL NUMBER OF TESTS
15
.??
POPULATION RELIABILITY

Conclusions:

Data Analysis:

- **Weibull Distributions due to its "flexibility" can be used in many situations to analyze test data.**
- **Weibull parameters include:**
- **Shape parameter, β: the slope of the weibull cumulative distribution function.**
- **Scale parameter, η: the "compression" of the weibull probability density function.**
- **Location parameter, γ: the "x" intercept of the weibull probability density function.**

Suspended Testing

- **For each failed item calculate the *mean order number*, i_{ti}, from: $i_{ti} = i_{ti-1} + N_{ti}$ where $N_{ti} = [(n+1)-i_{ti-1}]/[1+(n - \text{number of items beyond present suspended item})]$**

Conclusions (continued):

Confidence Limits:

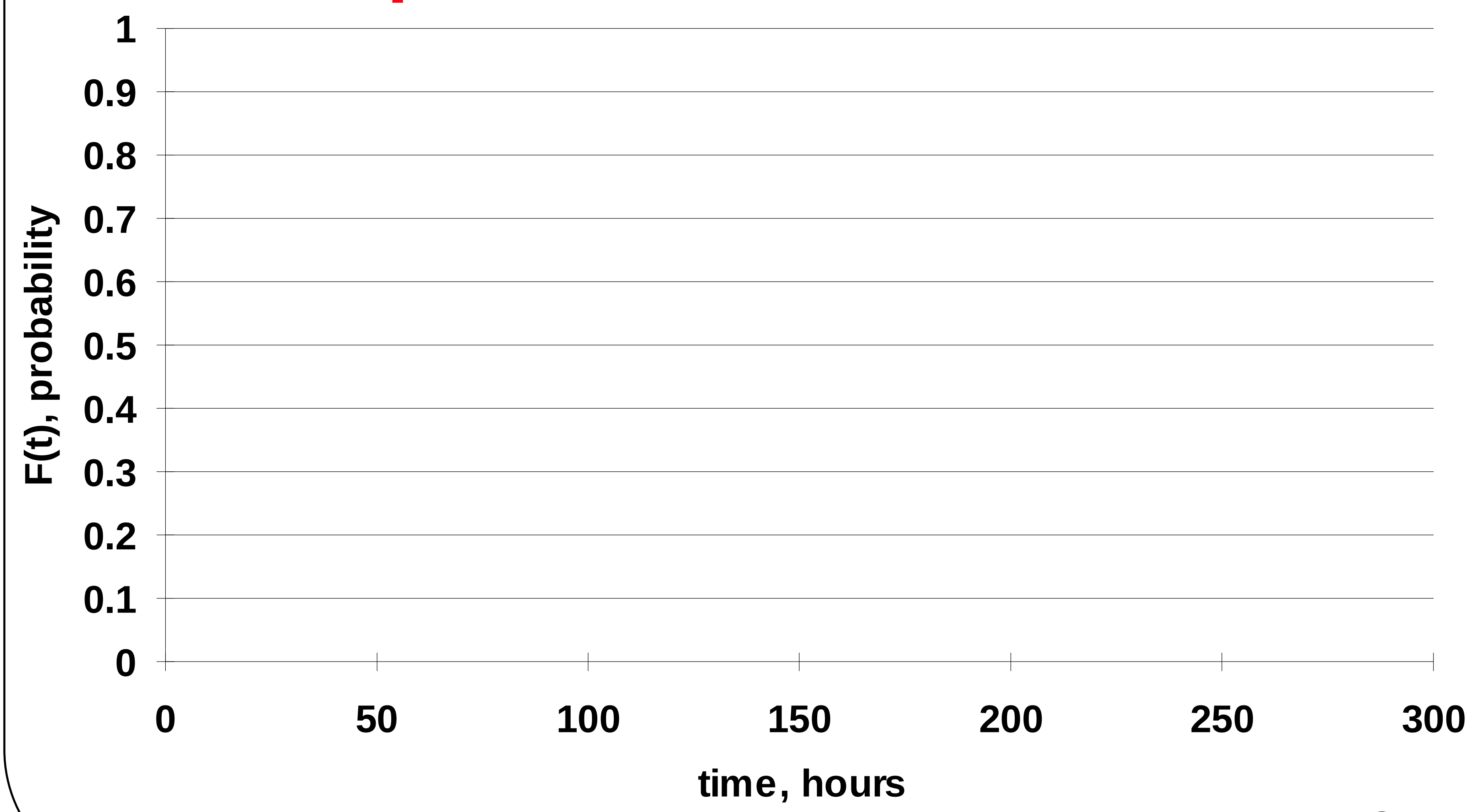
- **Help describe the level of certainty of data.**
- **Care must be taken to use them properly.**

Example: **The failure time of 10 CONTROL SHUTTLES (CS-114b) are recorded to be 110, 170, 205, 230, 260, 295, 325, 360, 400, 460 hrs. Use $F(t_i) = (i-0.3)/(n+0.4)$.**

i	*t*	$F(t_i)$	$R(t_i)$	$h(t)$	
1	110	______	______	______	______
2	170	______	______	______	______
3	205	______	______	______	______
4	______	______	______	______	______
5	______	______	______	______	______
6	______	______	______	______	______
7	______	______	______	______	______
8	______	______	______	______	______
9	______	______	______	______	______
10	______	______	______	______	______

F; P12-4 p. 1

Example (con't) -- PLOT CDF & RF



F; P12-4 p. 2

PLOT ON WEIBULL PROBABILITY PAPER

F; P12-4 p.3

DATA ANALYSIS-SUSPENDED TESTS

Assume individual data (Not grouped data).
Some units did not fail (Suspended Testing) & some were withdrawn.

- **List order number, *i*, of failed items in order of increasing life from sample size n.**
- **List suspended items (in order of suspended time) and combine with failed items.**
- **For each failed item calculate the *mean order number*, i_{ti}, from:**

$$i_{ti} = i_{ti-1} + N_{ti}$$

where $N_{ti} = [(n+1)-i_{ti-1}]/[1+(n - \text{number of items beyond present suspended item})]$

- ***Calculate the median rank for each failed item using:*** $F(t_i) = (i_{ti} - 0.3)/(n+0.4)$

DATA ANALYSIS--Suspended Testing (continued)

- Plot the data: failure times vs. $F(t_i)$ on graph paper with the x-axis as time (or cycles to failure) and the y-axis as cumulative percent (or probability) failure.
- Draw a line of best fit through the points.
- Calculate $R(t)$ and $h(t)$ and plot on regular graph paper (optional).
- Plot failure time vs. $F(t_i)$ *on* Weibull Probability Paper.

DATA ANALYSIS--Suspended Testing (continued)

- **Where:**
- ***$F(t_i)$ =** cumulative distribution function*
- ***i =** component number (rank order)*
- ***i_{ti} =** mean order number*
- ***N_{ti} =** number for intermediate calc.*
- ***n =** total number of samples*
- ***$R(t)$ =** reliability distribution function*
- ***$f(t)$ =** probability distribution function*
- ***$h(t)$ =** hazard function*